UNIT – I

Introduction to classical control system

- **System**

A System is a combination or an arrangement of different physical components which act together as an entire unit to achieve certain objective

- **Control System**

A control system is a system of devices or set of devices, that manages, commands, directs or regulates the behaviour of other device(s) or system(s) to achieve desire results.

A control system is a system, which controls other system.

- The main **feature of control system** is, there should be a clear mathematical relation between input and output of the system.
- When the relation between input and output of the system can be represented by a linear proportionality, the system is called linear control system.
- when the relation between input and output cannot be represented by single linear proportionality, rather the input and output are related by some non-linear relation, the system is referred as non-linear control system.

- **Requirement of Good Control System**

  - **Accuracy :** Accuracy is the measurement tolerance of the instrument and defines the limits of the errors made when the instrument is used in normal operating conditions. Accuracy can be improved by using feedback elements. To increase accuracy of any control system error detector should be present in control system.

  - **Sensitivity :** The parameters of control system are always changing with change in surrounding conditions, internal disturbance or any other parameters. This change can be expressed in terms of sensitivity. Any control system should be insensitive to such parameters but sensitive to input signals only.
  - **Noise :** An undesired input signal is known as noise. A good control system should be able to reduce the noise effect for better performance.
  - **Stability :** It is an important characteristic of control system. For the bounded input signal, the output must be bounded and if input is zero

then output must be zero then such a control system is said to be stable system.

- **Bandwidth :** An operating frequency range decides the bandwidth of control system. Bandwidth should be large as possible for frequency response of good control system.
- **Speed :** It is the time taken by control system to achieve its stable output. A good control system possesses high speed. The transient period for such system is very small.
- **Oscillation :** A small numbers of oscillation or constant oscillation of output tend to system to be stable.

Control Systems can be classified as open loop control systems and closed loop control systems based on the **feedback path**.

Open Loop Control System

A control system in which the control action is totally independent of output of the system then it is called **open loop control system**.

In **open loop control systems**, output is not fed-back to the input. So, the control action is independent of the desired output.

Manual control system is also an open loop control system.

The following figure shows the block diagram of the open loop control system.

Here, an input is applied to a controller and it produces an actuating signal or controlling signal. This signal is given as an input to a plant or process which is to be controlled. So, the plant produces an output, which is controlled. The traffic lights control system which we discussed earlier is an example of an open loop control system.

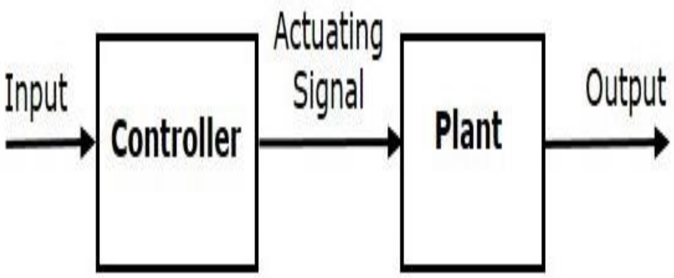


Figure 1.1: Block diagram of open loop control system

❖ Practical Examples of Open Loop Control System

1. **Electric Hand Drier -** Hot air (output) comes out as long as you keep your hand under the machine, irrespective of how much your hand is dried.
2. **Automatic Washing Machine -** This machine runs according to the pre-set time irrespective of washing is completed or not.
3. **Bread Toaster -** This machine runs as per adjusted time irrespective of toasting is completed or not.
4. **Automatic Tea/Coffee Maker -** These machines also function for pre adjusted time only.
5. **Timer Based Clothes Drier -** This machine dries wet clothes for pre-adjusted time, it does not matter how much the clothes are dried.
6. **Light Switch -** Lamps glow whenever light switch is on irrespective of light is required or not.
7. **Volume on Stereo System -** Volume is adjusted manually irrespective of output volume level.

❖ Advantages of Open Loop Control System

1. Simple in construction and design.
2. Economical.
3. Easy to maintain.
4. Generally stable.
5. Convenient to use as output is difficult to measure.

❖ Disadvantages of Open Loop Control System

1. They are inaccurate.
2. They are unreliable.
3. Any change in output cannot be corrected automatically.

➢ Closed Loop Control System

Control system in which the output has an effect on the input quantity in such a manner that the input quantity will adjust itself based on the output generated is called **closed loop control system**.

In **closed loop control systems**, output is fed back to the input. So, the control action is dependent on the desired output.

The following figure shows the block diagram of negative feedback closed loop control system.

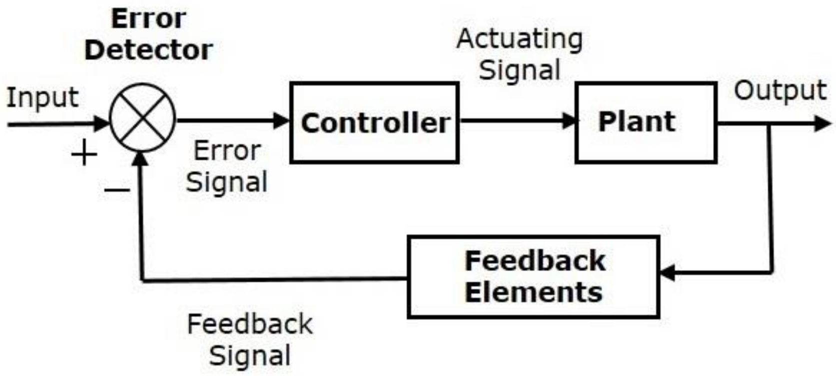


Figure 1.2:Block diagram of closed loop control system

The error detector produces an error signal, which is the difference between the input and the feedback signal. This feedback signal is obtained from the block (feedback elements) by considering the output of the overall system as an input to this block. Instead of the direct input, the error signal is applied as an input to a controller.

So, the controller produces an actuating signal which controls the plant. In this combination, the output of the control system is adjusted automatically till we get the desired response. Hence, the closed loop control systems are also called the automatic control systems. Traffic lights control system having sensor at the input is an example of a closed loop control system.

❖ **Practical Examples of Closed Loop Control System**

1. **Automatic Electric Iron** - Heating elements are controlled by output temperature of the iron.
2. **Servo Voltage Stabilizer** - Voltage controller operates depending upon output voltage of the system.
3. **Water Level Controller** - Input water is controlled by water level of the reservoir.
4. **Missile Launched and Auto Tracked by Radar** - The direction of missile is controlled by comparing the target and position of the missile.

5. **An Air Conditioner** - An air conditioner functions depending upon the temperature of the room.
6. **Cooling System in Car** - It operates depending upon the temperature which it controls.

❖ Advantages of Closed Loop Control System

1. Closed loop control systems are more accurate even in the presence of non-linearity.
2. Highly accurate as any error arising is corrected due to presence of feedback signal.
3. Bandwidth range is large.
4. Facilitates automation.
5. The sensitivity of system may be made small to make system more stable.
6. This system is less affected by noise.

❖ Disadvantages of Closed Loop Control System

1. They are costlier.
2. They are complicated to design.
3. Required more maintenance.
4. Feedback leads to oscillatory response.
5. Overall gain is reduced due to presence of feedback.
6. Stability is the major problem and more care is needed to design a stable closed loop system.

➢ **Comparison of Closed Loop And Open Loop Control System**

Sr. No.	Open loop control system	Closed loop control system
1	The feedback element is absent.	The feedback element is always present.
2	An error detector is not present.	An error detector is always present.
3	It is stable one.	It may become unstable.
4	Easy to construct.	Complicated construction.
5	It is an economical.	It is costly.
6	Having small bandwidth.	Having large bandwidth.
7	It is inaccurate.	It is accurate.
8	Less maintenance.	More maintenance.
9	It is unreliable.	It is reliable.
10	Examples: Hand drier, tea maker	Examples: Servo voltage stabilizer, perspiration

➢ **Feedback**

If either the output or some part of the output is returned to the input side and utilized as part of the system input, then it is known as **feedback**. Feedback plays an important role in order to improve the performance of the control systems. In this chapter, let us discuss the types of feedback & effects of feedback.

Types of Feedback

There are two types of feedback −

- Positive feedback
- Negative feedback

Positive Feedback

The positive feedback adds the reference input, R(s) and feedback output. The following figure shows the block diagram of **positive feedback control system**.

The concept of transfer function will be discussed in later chapters. For the time being, consider the transfer function of positive feedback control system is,

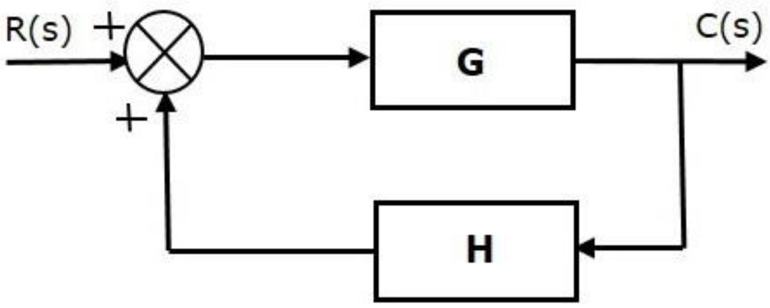


Figure 1.3: Block diagram of positive feedback system

T=G/(1−GH) (Equation 1)
Where,

- **T** is the transfer function or overall gain of positive feedback control system.
- **G** is the open loop gain, which is function of frequency.
- **H** is the gain of feedback path, which is function of frequency.

Negative Feedback

Negative feedback reduces the error between the reference input, R(s)R(s) and system output. The following figure shows the block diagram of the **negative feedback control system**.

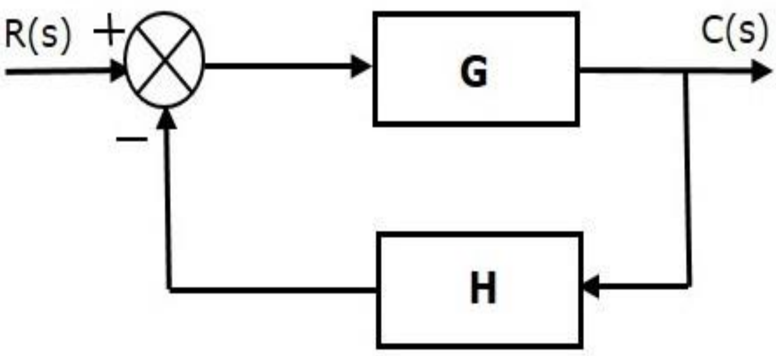


Figure 1.4: Block diagram of negative feedback system

Transfer function of negative feedback control system is,

T=G/(1+GH) (Equation 2)

Effects of Feedback

Let us now understand the effects of feedback.

Effect of Feedback on Overall Gain

- From Equation 2, we can say that the overall gain of negative feedback closed loop control system is the ratio of 'G' and (1+GH). So, the overall gain may increase or decrease depending on the value of (1+GH).

- If the value of (1+GH) is less than 1, then the overall gain increases. In this case, 'GH' value is negative because the gain of the feedback path is negative.

- If the value of (1+GH) is greater than 1, then the overall gain decreases. In this case, 'GH' value is positive because the gain of the feedback path is positive.

In general, 'G' and 'H' are functions of frequency. So, the feedback will increase the overall gain of the system in one frequency range and decrease in the other frequency range.

Effect of Feedback on Sensitivity

Sensitivity of the overall gain of negative feedback closed loop control system (**T**) to the variation in open loop gain (**G**) is defined as $S_G^T = \frac{\frac{\delta T}{T}}{\frac{\delta G}{G}}$ (equation 3)

Where, **∂T** is the incremental change in T due to incremental change in G.

We can rewrite Equation 3 as $S_G^T = \frac{\delta T}{\delta G}\frac{G}{T}$ (equation 4)

Do partial differentiation with respect to G on both sides of Equation 2.

$$\frac{\partial T}{\partial G} = \frac{\partial}{\partial G}\left(\frac{G}{1+GH}\right) = \frac{(1+GH).1-G(H)}{(1+GH)^2} = \frac{1}{(1+GH)^2} \qquad \text{(Equation 5)}$$

From Equation 2, you will get

$$\frac{G}{T} = 1+GH \qquad \text{(equation 6)}$$

Substitute Equation 5 and Equation 6 in Equation 4.

$$S_G^T = \frac{1}{(1+GH)^2}(1+GH) = \frac{1}{1+GH}$$

So, we got the **sensitivity** of the overall gain of closed loop control system as the reciprocal of (1+GH). So, Sensitivity may increase or decrease depending on the value of (1+GH).

- If the value of (1+GH) is less than 1, then sensitivity increases. In this case, 'GH' value is negative because the gain of feedback path is negative.

- If the value of (1+GH) is greater than 1, then sensitivity decreases. In this case, 'GH' value is positive because the gain of feedback path is positive.

In general, 'G' and 'H' are functions of frequency. So, feedback will increase the sensitivity of the system gain in one frequency range and decrease in the other frequency range. Therefore, we have to choose the values of 'GH' in such a way that the system is insensitive or less sensitive to parameter variations.

Effect of Feedback on Stability

- A system is said to be stable, if its output is under control. Otherwise, it is said to be unstable.

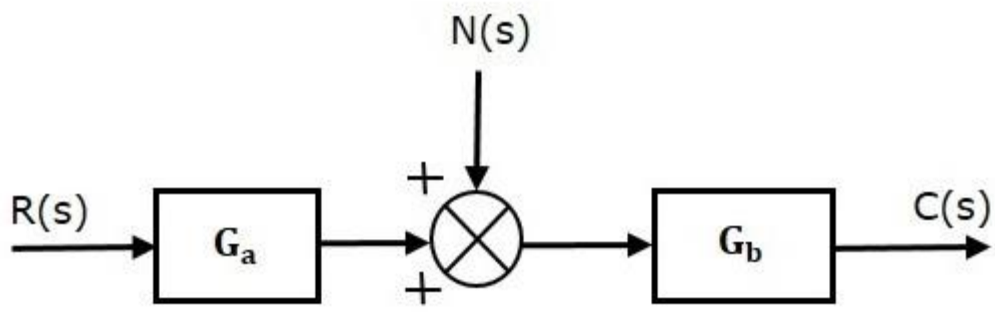
- In Equation 2, if the denominator value is zero (i.e., GH = -1), then the output of the control system will be infinite. So, the control system becomes unstable.

Therefore, we have to properly choose the feedback in order to make the control system stable.

Effect of Feedback on Noise

To know the effect of feedback on noise, let us compare the transfer function relations with and without feedback due to noise signal alone.

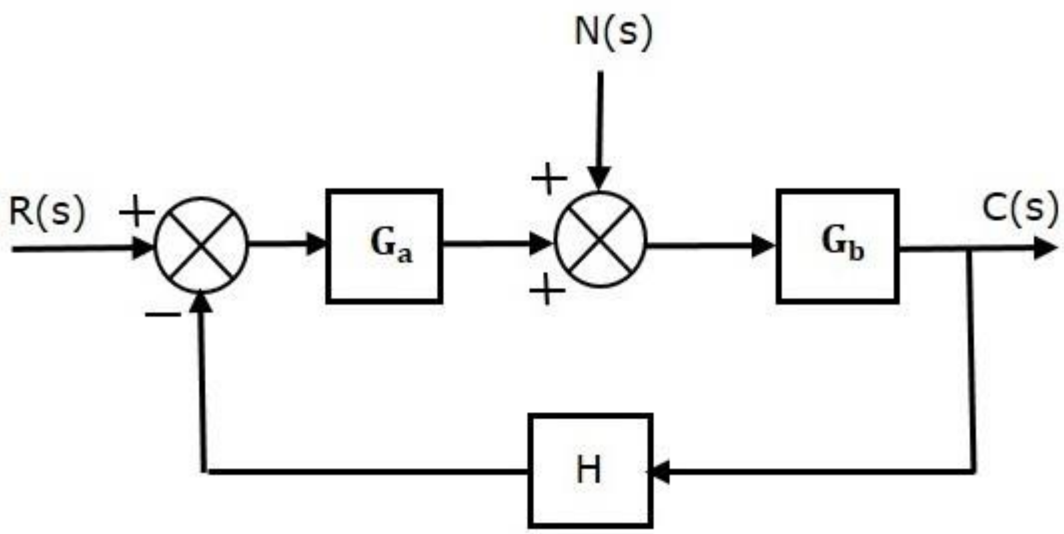
Consider an **open loop control system** with noise signal as shown below.



The **open loop transfer function** due to noise signal alone is

$$\frac{C(s)}{N(s)} = G_b \qquad \text{(Equation 7)}$$

It is obtained by making the other input R(s)R(s) equal to zero.
Consider a **closed loop control system** with noise signal as shown below.



The **closed loop transfer function** due to noise signal alone is

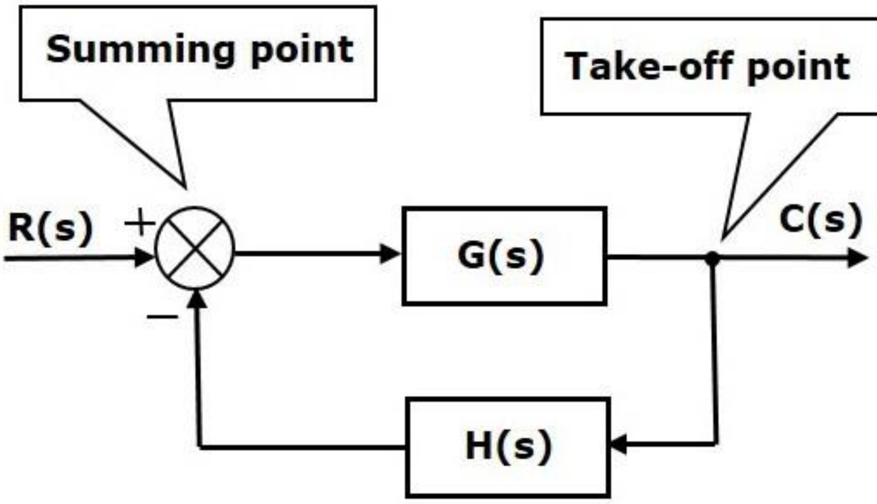
$$\frac{C(s)}{N(s)} = \frac{G_b}{1 + G_a G_b H} \qquad \text{(Equation 8)}$$

It is obtained by making the other input R(s) equal to zero.
Compare Equation 7 and Equation 8, In the closed loop control system, the gain due to noise signal is decreased by a factor of $(1+G_aG_bH)$ provided that the term $(1+G_aG_bH)$ is greater than one.

Block diagrams consist of a single block or a combination of blocks. These are used to represent the control systems in pictorial form.

Basic Elements of Block Diagram

The basic elements of a block diagram are a block, the summing point and the take-off point. Let us consider the block diagram of a closed loop control system as shown in the following figure to identify these elements.



The above block diagram consists of two blocks having transfer functions G(s) and H(s). It is also having one summing point and one take-off point. Arrows indicate the direction of the flow of signals. Let us now discuss these elements one by one.

Block

The transfer function of a component is represented by a block. Block has single input and single output.

The following figure shows a block having input X(s), output Y(s) and the transfer function G(s).

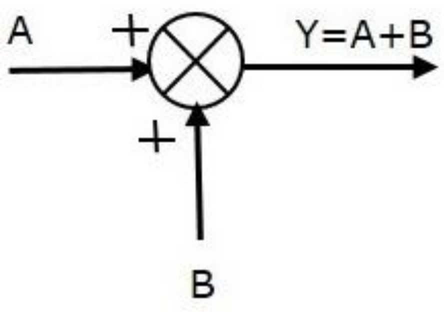
Transfer Function, $G(s) = \frac{Y(s)}{X(s)}$ $\Rightarrow Y(s) = G(s)X(s)$

Output of the block is obtained by multiplying transfer function of the block with input.

Summing Point

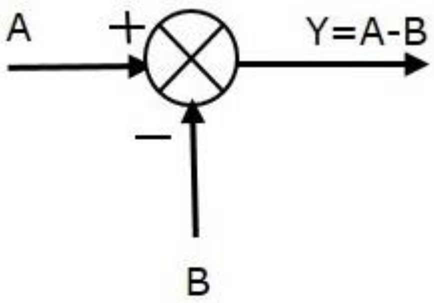
The summing point is represented with a circle having cross (X) inside it. It has two or more inputs and single output. It produces the algebraic sum of the inputs. It also performs the summation or subtraction or combination of summation and subtraction of the inputs based on the polarity of the inputs. Let us see these three operations one by one.

The following figure shows the summing point with two inputs (A, B) and one output (Y). Here, the inputs A and B have a positive sign. So, the summing point produces the output, Y as **sum of A and B.**



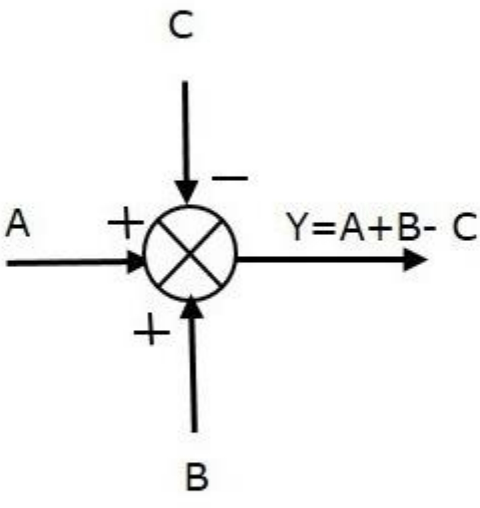
i.e.,Y = A + B.

The following figure shows the summing point with two inputs (A, B) and one output (Y). Here, the inputs A and B are having opposite signs, i.e., A is having positive sign and B is having negative sign. So, the summing point produces the output **Y** as the **difference of A and B**.



Y = A + (-B) = A - B.

The following figure shows the summing point with three inputs (A, B, C) and one output (Y). Here, the inputs A and B are having positive signs and C is having a negative sign. So, the summing point produces the output **Y** as

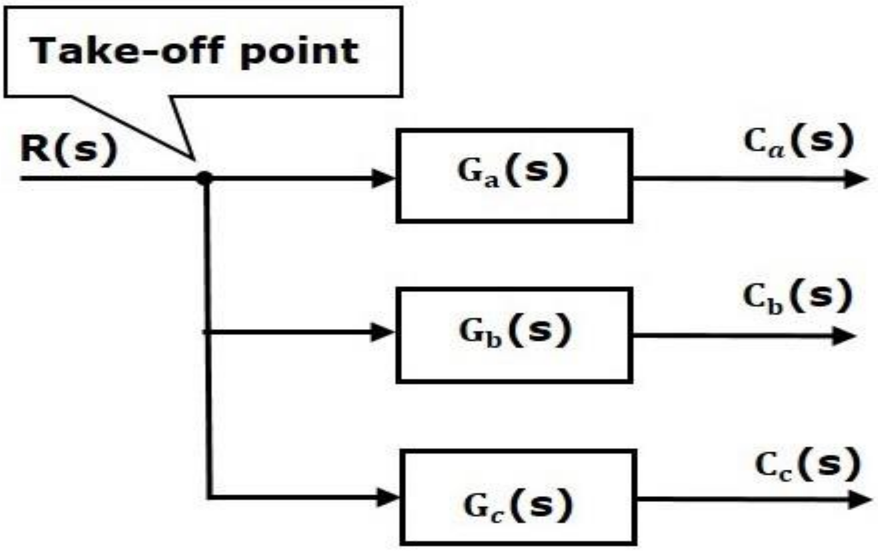


Y = A + B + (−C) = A + B − C.

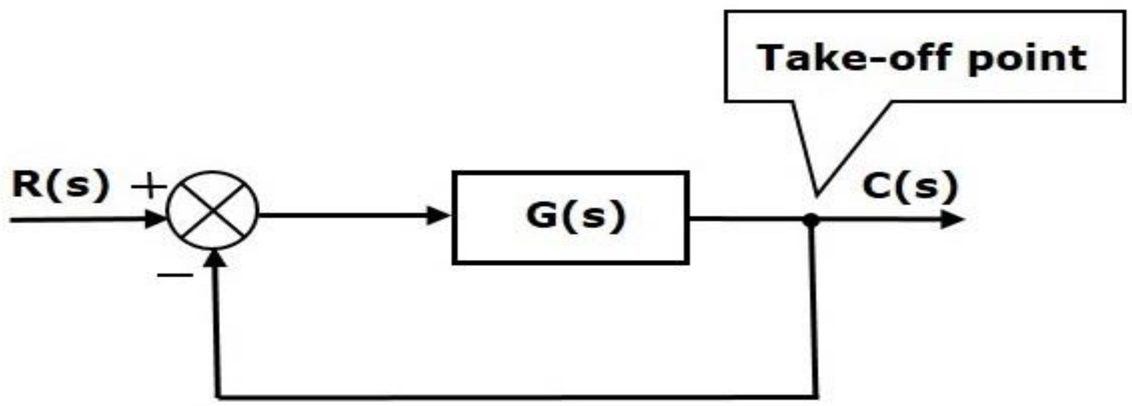
Take-off Point

The take-off point is a point from which the same input signal can be passed through more than one branch. That means with the help of take-off point, we can apply the same input to one or more blocks, summing points.

In the following figure, the take-off point is used to connect the same input, R(s) to two more blocks.



In the following figure, the take-off point is used to connect the output C(s), as one of the inputs to the summing point.



Disadvantages of Block Diagram Representation

- No information about the physical construction
- Source of energy is not shown

Advantages of Block Diagram Representation

- Very simple to construct block diagram for a complicated system
- Function of individual element can be visualized
- Individual & Overall performance can be studied
- Over all transfer function can be calculated easily.

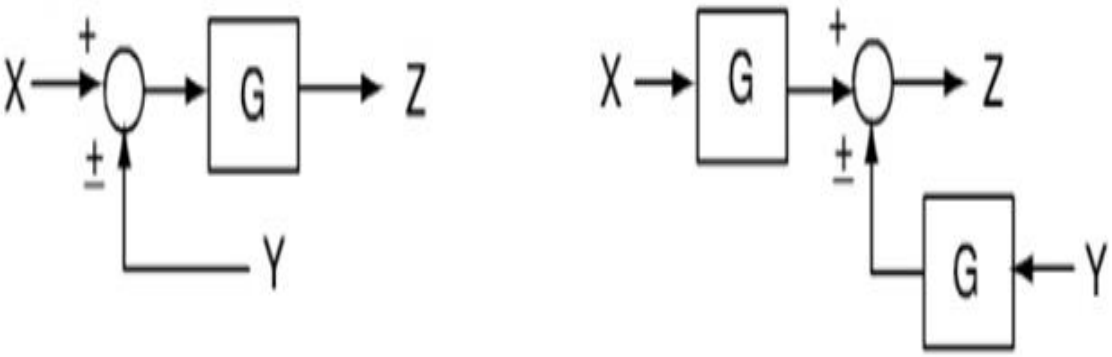
Block diagram reduction technique

Because of their simplicity and versatility, block diagrams are often used by control engineers to describe all types of systems. A block diagram can be used simply to represent the composition and interconnection of a system. Also, it can be used, together with transfer functions, to represent the cause-and-effect relationships throughout the system. Transfer Function is defined as the relationship between an input signal and an output signal to a device.

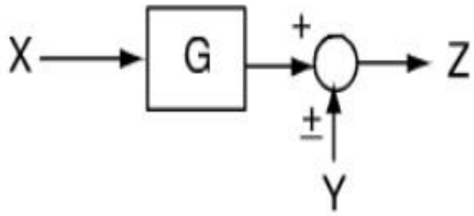
Block diagram rules

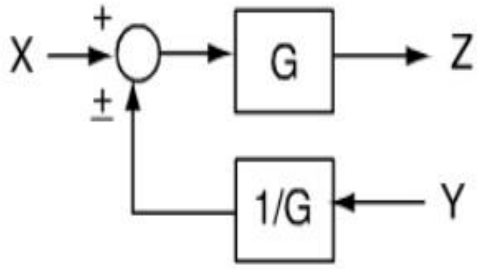
Cascaded blocks

Moving a summer beyond the block

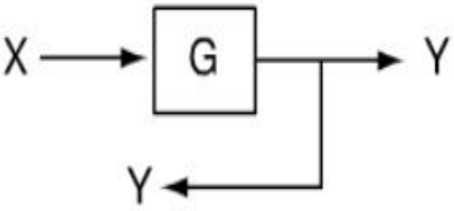


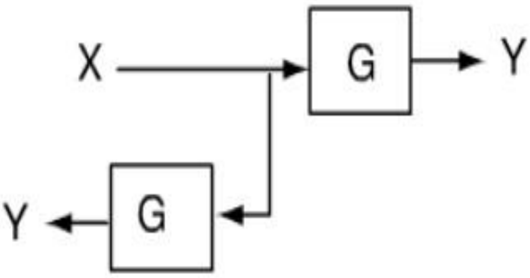
Moving a summer ahead of block



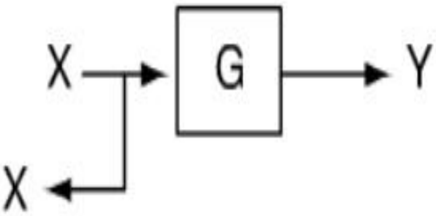


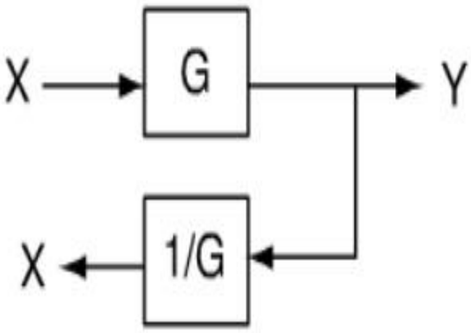
Moving a pick-off ahead of block



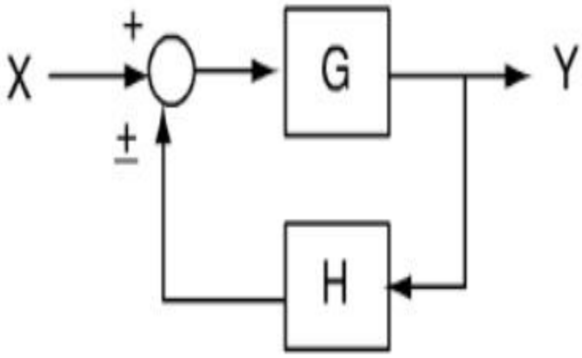


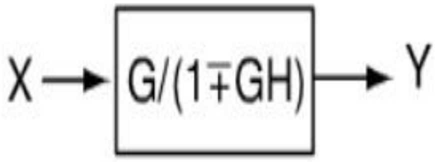
Moving a pick-off behind a block



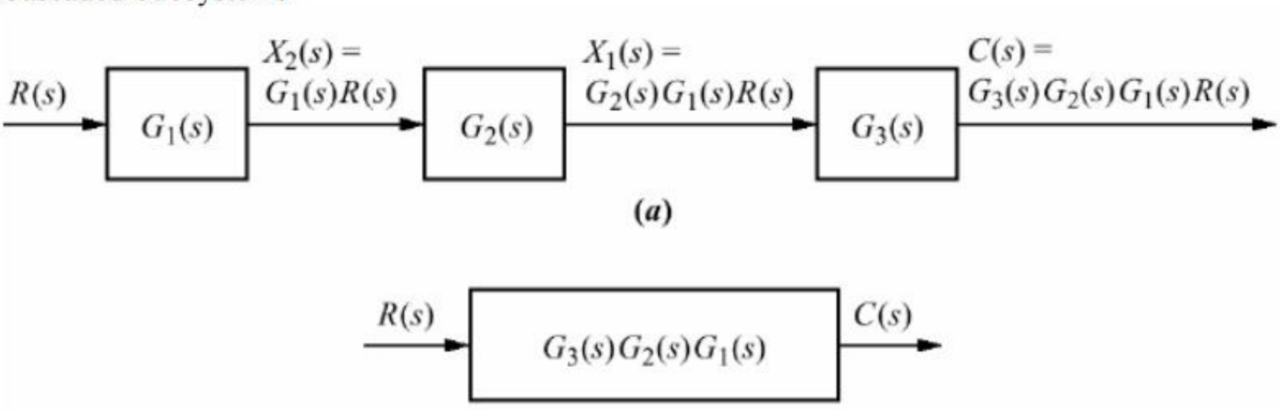


Eliminating a feedback loop

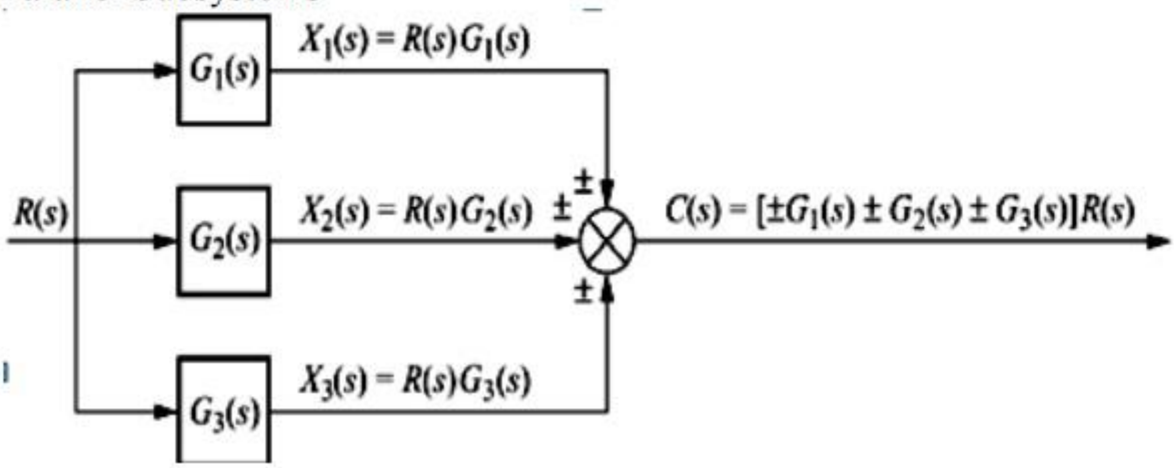




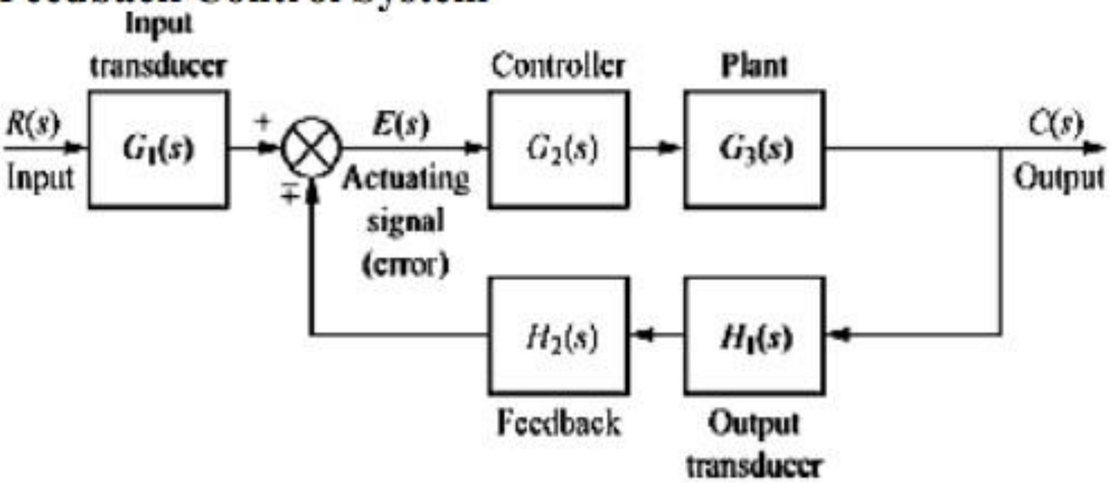
Cascaded Subsystems



Parallel Subsystems



Feedback Control System



Procedure to solve Block Diagram Reduction Problems

Step 1: Reduce the blocks connected in series

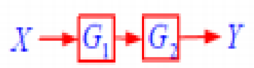
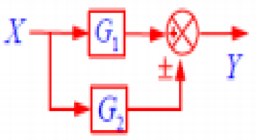
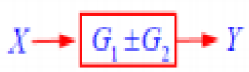
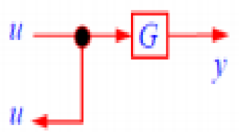
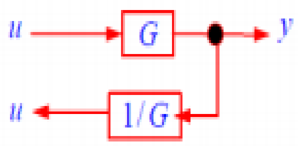
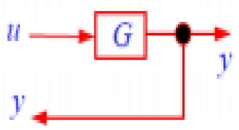
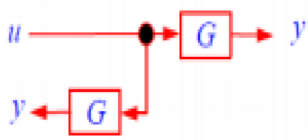
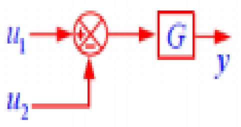
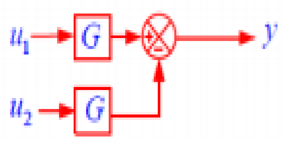
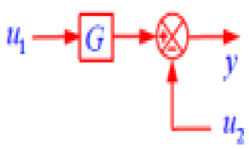
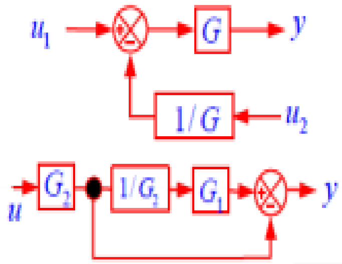
Step2: Reduce the blocks connected in parallel

Step 3: Reduce the minor feedback loops

Step 4: Try to shift take off points towards right and Summing point towards left

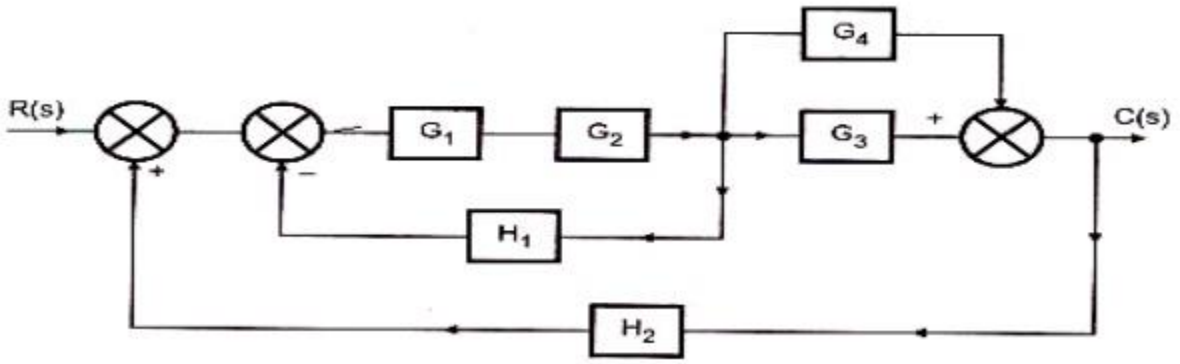
Step 5: Repeat steps 1 to 4 till simple form is obtained

Step 6: Obtain the Transfer Function of Overall System

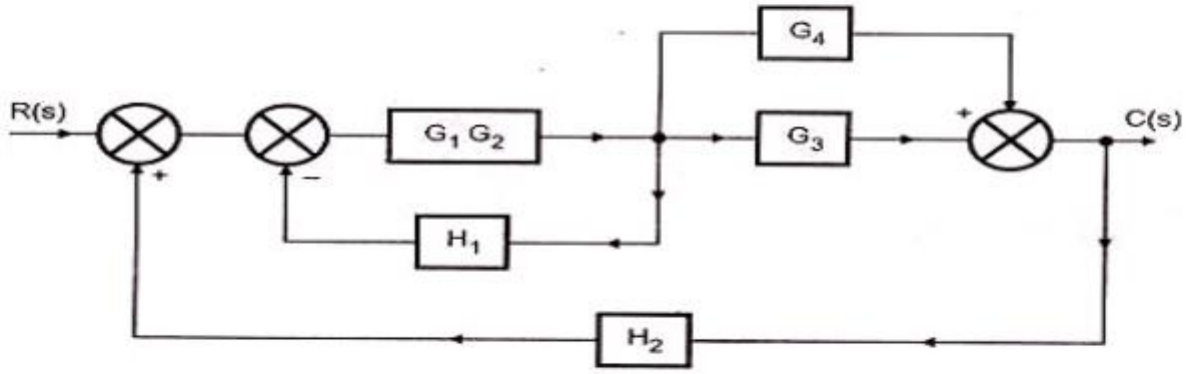
	Manipulation	Original Block Diagram	Equivalent Block Diagram	Equation
1	Combining Blocks in Cascade			$Y=(G_1G_2)X$
2	Combining Blocks in Parallel; or Eliminating a Forward Loop			$Y=(G_1\pm G_2)X$
3	Moving a pickoff point behind a block			$y = Gu$ $u = \frac{1}{G}y$
4	Moving a pickoff point ahead of a block			$y=Gu$
5	Moving a summing point behind a block			$e_2=G(u_1-u_2)$
6	Moving a summing point ahead of a block			$y=Gu_1-u_2$ $y=(G_1-G_2)u$

Problem 1

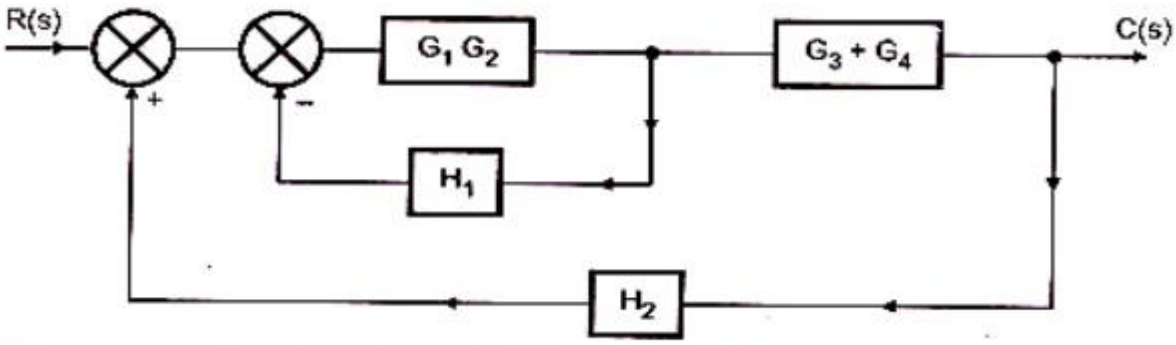
1. Obtain the Transfer function of the given block diagram



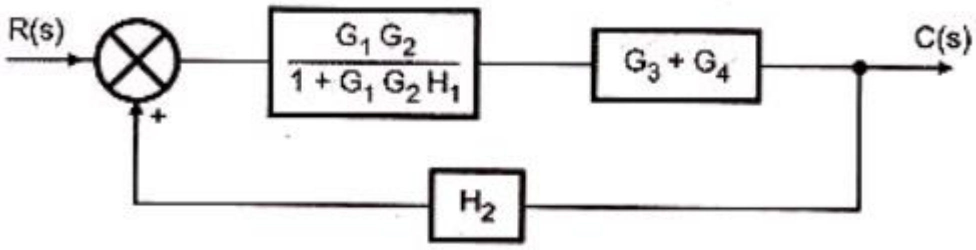
Combine G1, G2 which are in series

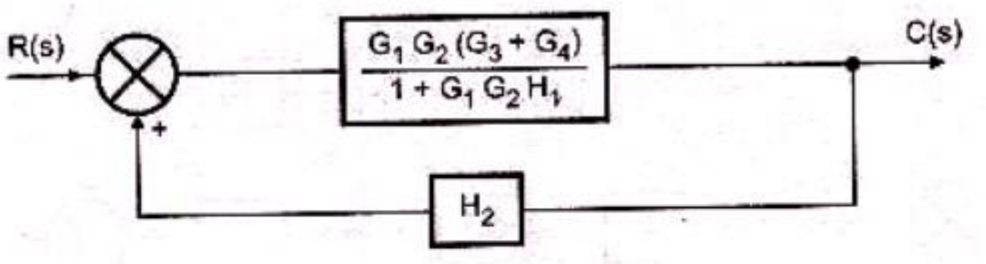


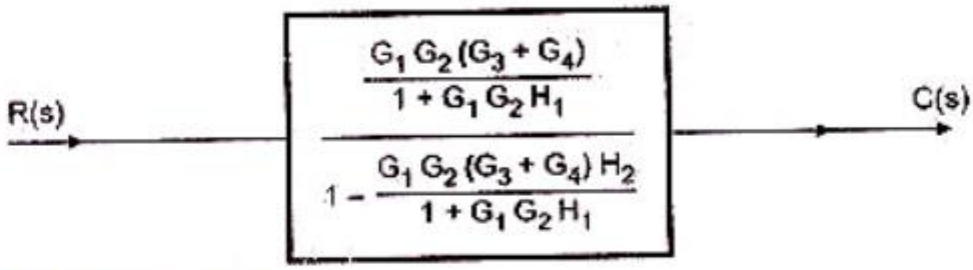
Combine G3, G4 which are in Parallel



Reduce minor feedback loop of G1, G2 and H1



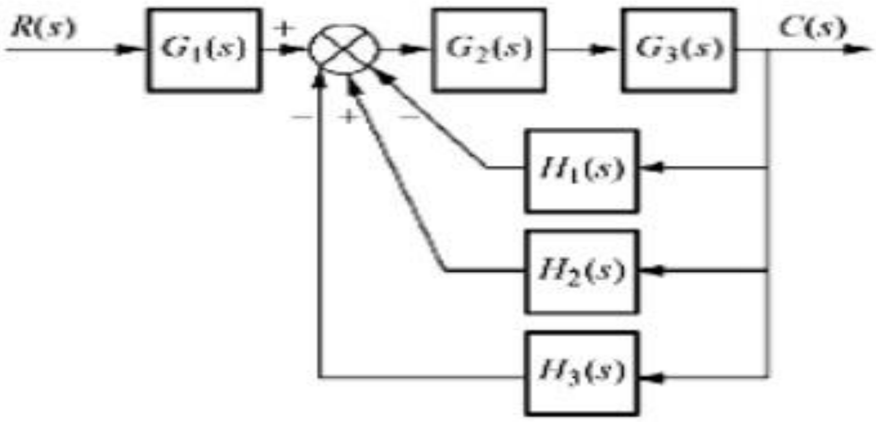




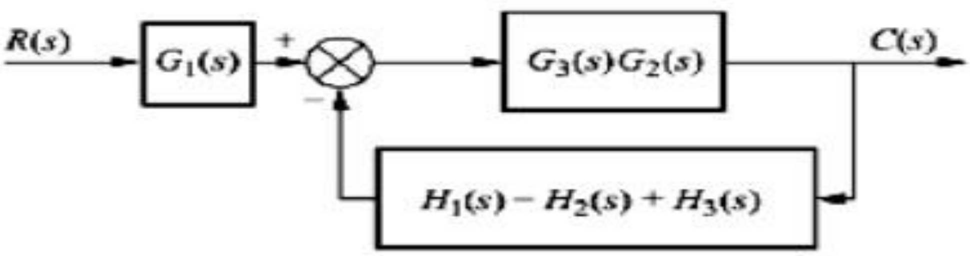
Transfer function

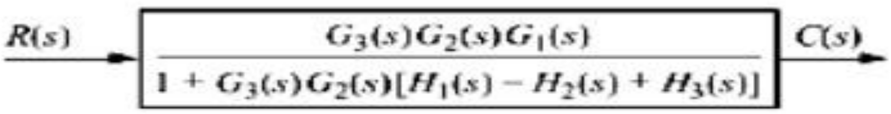
$$\frac{C(s)}{R(s)} = \frac{G_1 G_2 (G_3 + G_4)}{1 + G_1 G_2 H_1 - G_1 G_2 (G_3 + G_4) H_2}$$

2. Obtain the transfer function for the system shown in the fig

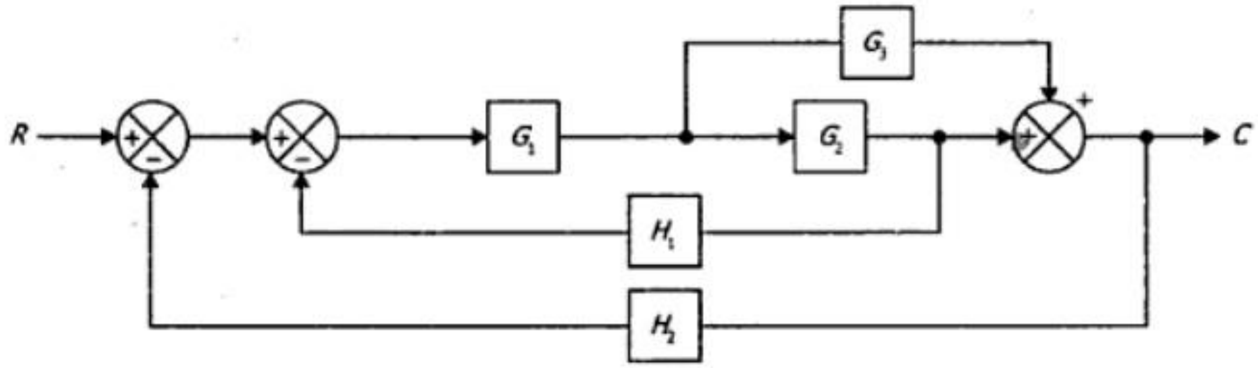


Solution





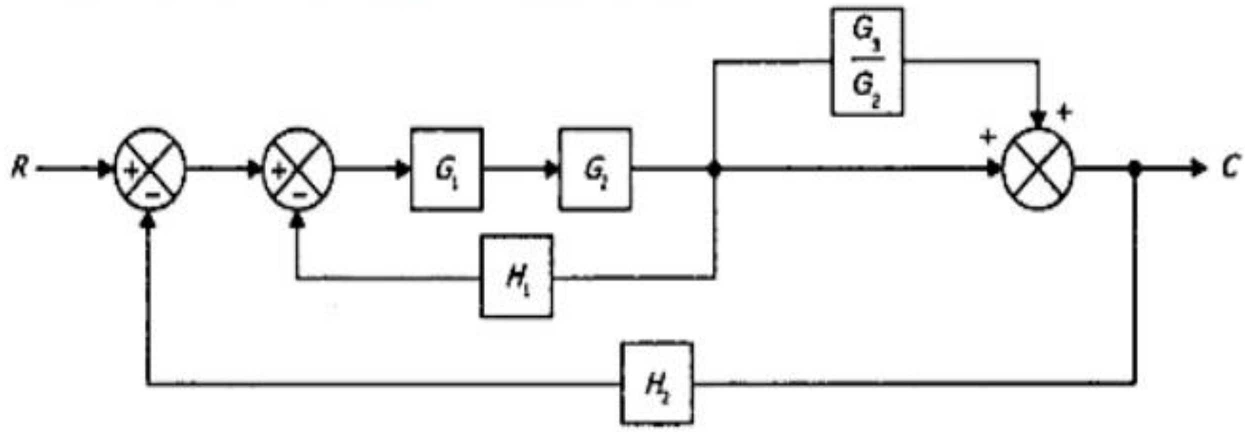
3. Obtain the transfer function C/R for the block diagram shown in the fig



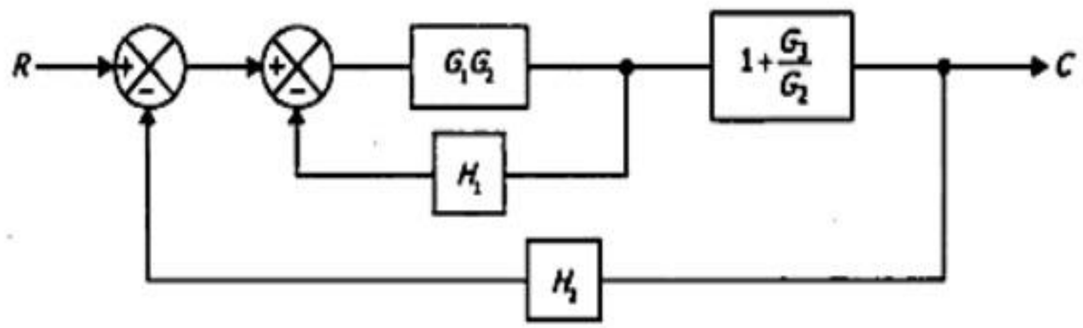
Solution

The take-off point is shifted after the block G2

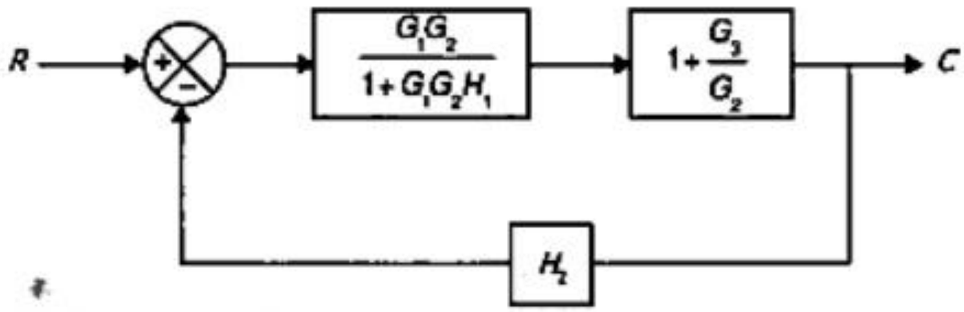
The take-off point is shifted after the block G2



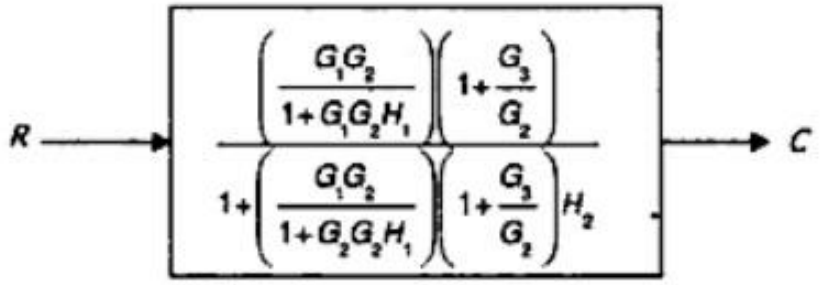
Reducing the cascade block and parallel block



Replacing the internal feedback loop



Equivalent block diagram

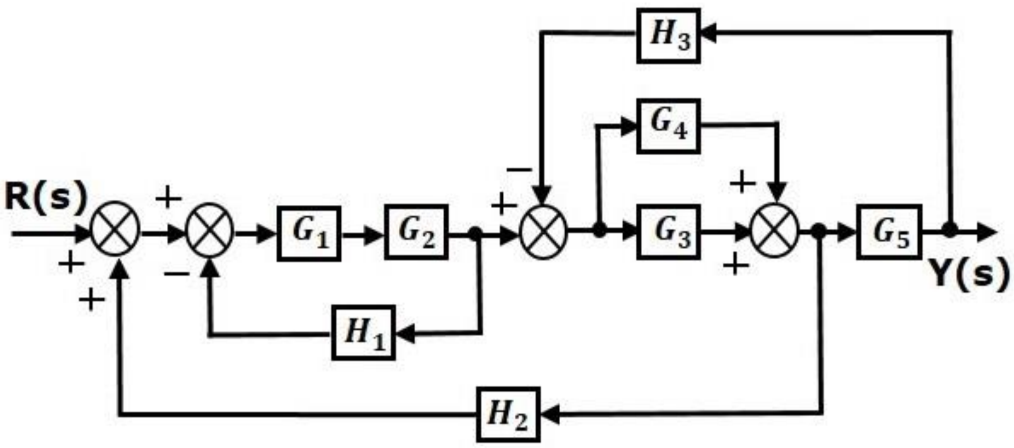


Transfer function

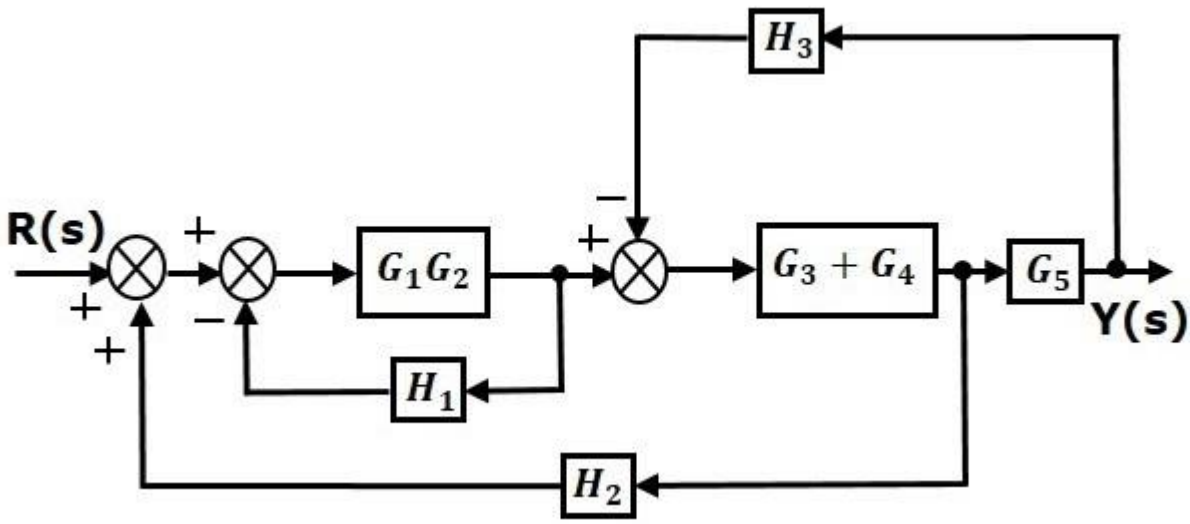
$$\frac{C}{R} = \frac{\dfrac{G_1(G_2+G_3)}{1+G_1G_2H_1}}{1+\dfrac{G_1(G_2+G_3)H_2}{1+G_1G_2H_1}}$$

$$= \frac{G_1(G_2+G_3)}{1+G_1G_2(H_1+H_2)+G_1G_3H_2}$$

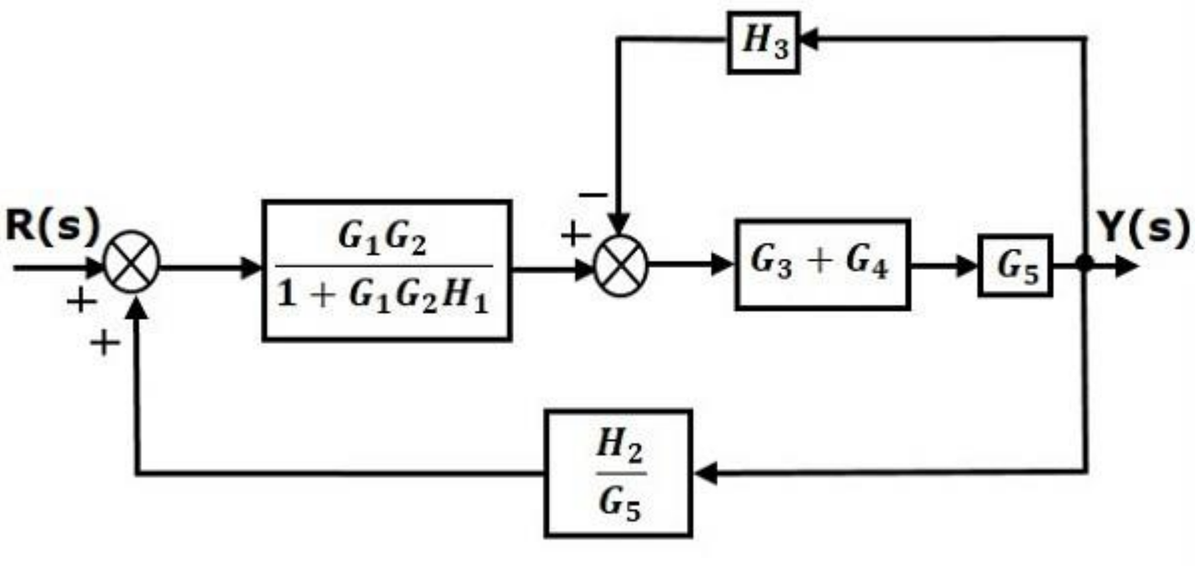
4. Obtain the transfer function C/R for the block diagram shown in the fig using the block diagram reduction rules.



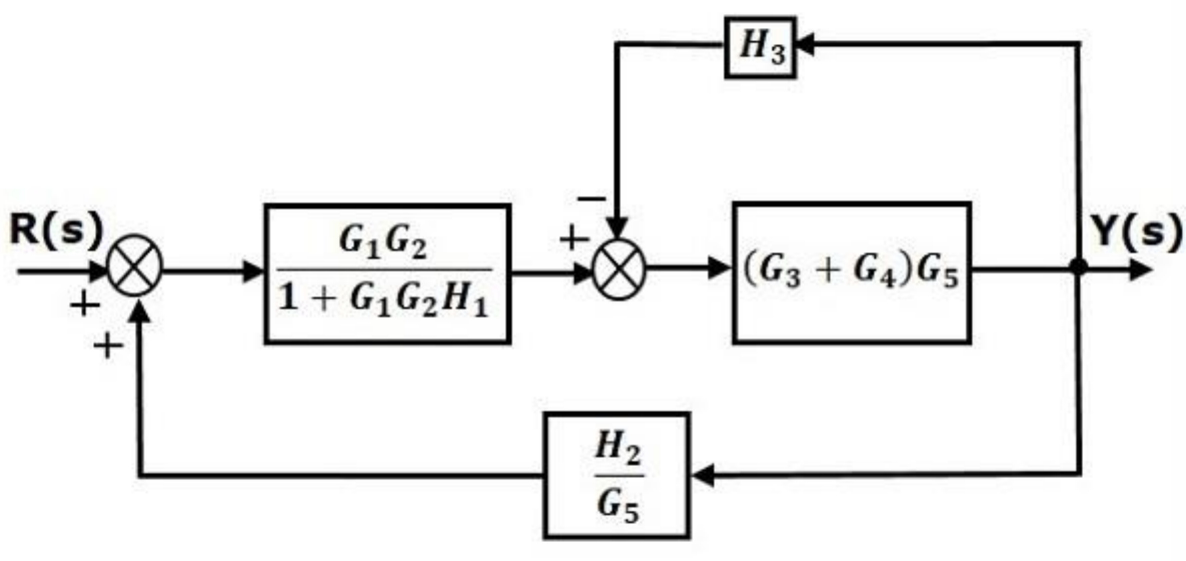
Step 1 – Use Rule 1 for blocks G_1 and G2. Use Rule 2 for blocks G3 and G4. The modified block diagram is shown in the following figure.



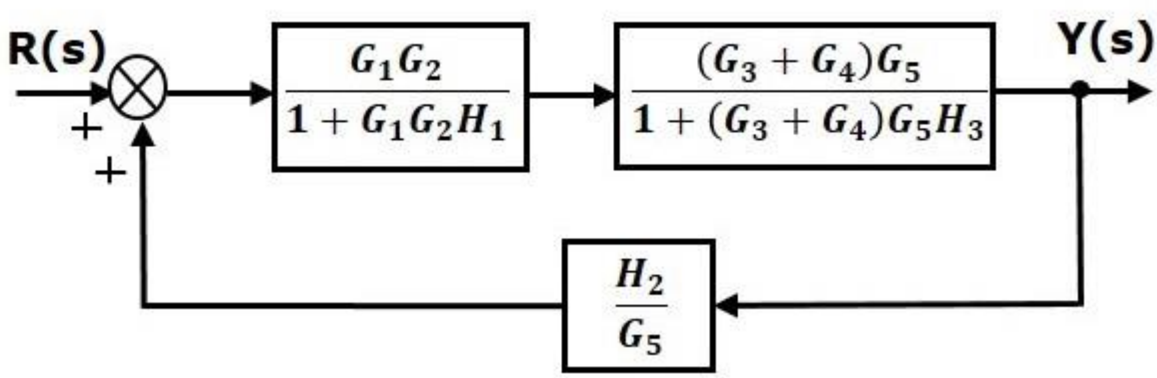
Step 2 – Use Rule 3 for blocks G_1G_2 and H_1. Use Rule 4 for shifting take-off point after the block G_5. The modified block diagram is shown in the following figure.



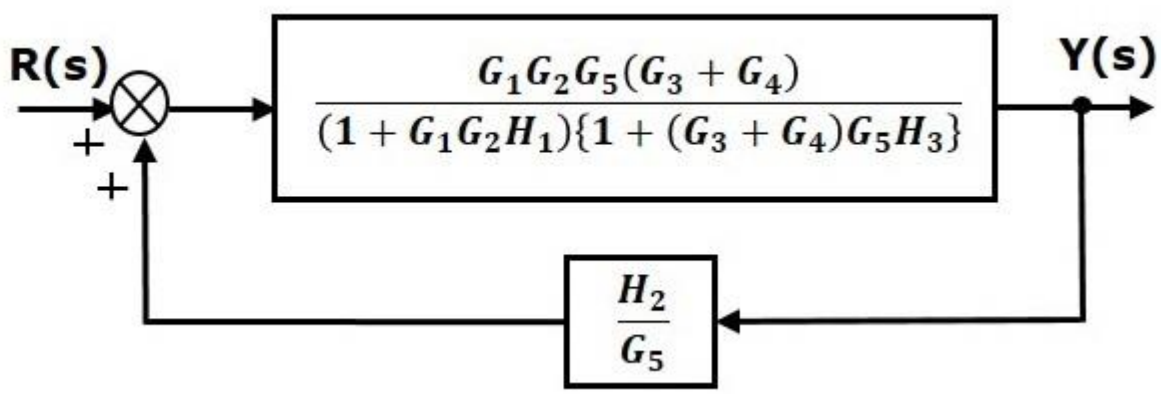
Step 3 – Use Rule 1 for blocks (G_3+G_4) and G_5. The modified block diagram is shown in the following figure.



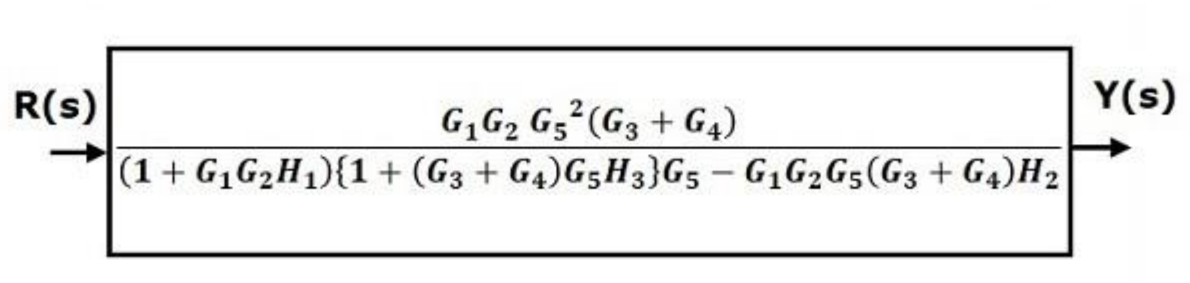
Step 4 – Use Rule 3 for blocks $(G_3+G_4)G_5$ and H_3. The modified block diagram is shown in the following figure.



Step 5 – Use Rule 1 for blocks connected in series. The modified block diagram is shown in the following figure.



Step 6 – Use Rule 3 for blocks connected in feedback loop. The modified block diagram is shown in the following figure. This is the simplified block diagram.



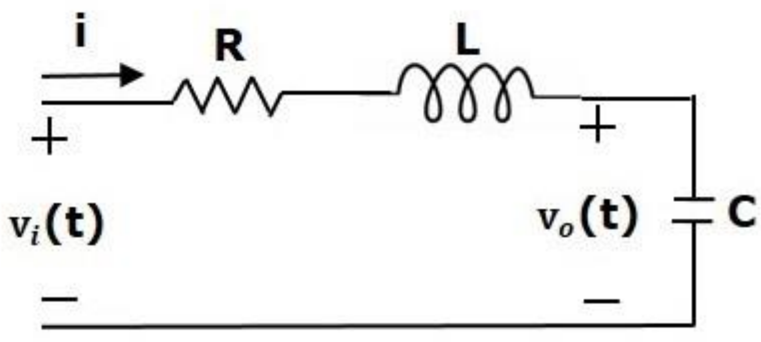
Therefore, the transfer function of the system is

$$\frac{Y(s)}{R(s)} = \frac{G_1G_2G_5^2(G_3+G_4)}{(1+G_1G_2H_1)(1+(G_3+G_4)G_5H_3)G_5 - G_1G_2G_5(G_3+G_4)H_2}$$

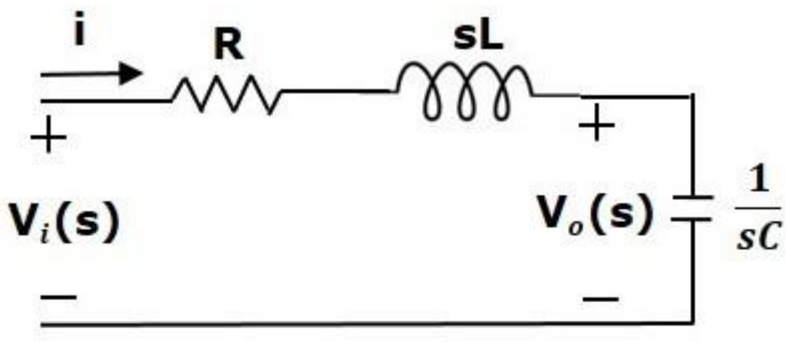
- **Block Diagram Representation of Electrical Systems**

In this section, let us represent an electrical system with a block diagram. Electrical systems contain mainly three basic elements — **resistor, inductor and capacitor**.

Consider a series of RLC circuit as shown in the following figure. Where, $V_i(t)$ and $V_o(t)$ are the input and output voltages. Let i(t) be the current passing through the circuit. This circuit is in time domain.



By applying the Laplace transform to this circuit, will get the circuit in s-domain. The circuit is as shown in the following figure.



From the above circuit, we can write

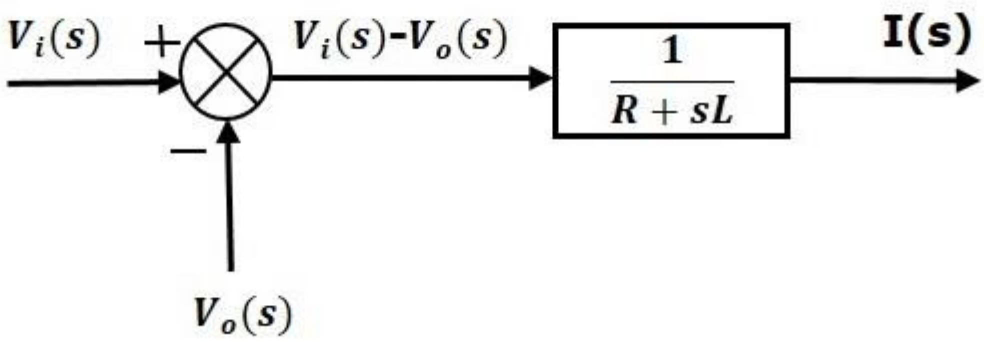
$$I(s) = \frac{V_i(s) - V_o(s)}{R + sL}$$

$$\Rightarrow I(s) = \left\{\frac{1}{R+sL}\right\}\{V_i(s) - V_o(s)\} \qquad \textbf{(Equation 1)}$$

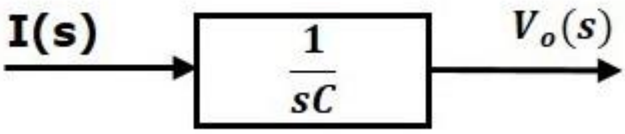
$$V_o(s) = \left(\frac{1}{sC}\right) I(s) \qquad \textbf{(Equation 2)}$$

Let us now draw the block diagrams for these two equations individually. And then combine those block diagrams properly in order to get the overall block diagram of series of RLC Circuit (s-domain).

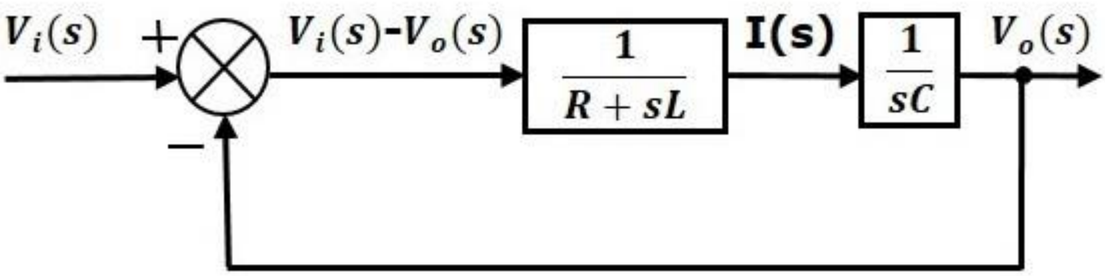
Equation 1 can be implemented with a block having the transfer function, $\frac{1}{R+SL}$ .The input and output of this block are $\{V_i(s)-V_o(s)\}$ and I(s). We require a summing point to get $\{V_i(s)-V_o(s)\}$. The block diagram of Equation 1 is shown in the following figure.



Equation 2 can be implemented with a block having transfer function, 1sC1sC. The input and output of this block are I(s)I(s) and Vo(s)Vo(s). The block diagram of Equation 2 is shown in the following figure.



The overall block diagram of the series of RLC Circuit (s-domain) is shown in the following figure.



Similarly, you can draw the **block diagram** of any electrical circuit or system just by following this simple procedure.

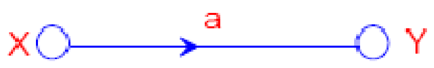
- Convert the time domain electrical circuit into an s-domain electrical circuit by applying Laplace transform.
- Write down the equations for the current passing through all series branch elements and voltage across all shunt branches.
- Draw the block diagrams for all the above equations individually.
- Combine all these block diagrams properly in order to get the overall block diagram of the electrical circuit (s-domain).

The block diagram reduction process takes more time for complicated systems. Because, we have to draw the (partially simplified) block diagram after each step. So, to overcome this drawback, use signal flow graphs (representation) i.e., how to represent signal flow graph from a given block diagram and calculation of transfer function just by using a gain formula without doing any reduction process.

➢ SIGNAL FLOW GRAPHS

For complex control systems, the block diagram reduction technique is cumbersome. An alternative method for determining the relationship between system variables has been developed by ***Mason*** and is based on a signal flow graph. A signal flow graph is a diagram that consists of nodes that are connected by branches. A node is assigned to each variable of interest in the system, and branches are used to relate the different variables. The main advantage for using SFG is that a straight forward procedure is available for finding the transfer function in which it is not necessary to move pickoff point around or to redraw the system several times as with block diagram manipulations.

SFG is a diagram that represents a set of simultaneous linear algebraic equations which describe a system. Let us consider an equation, y = a x. It may be represented graphically as,



Definitions:

Node: A node is a point representing a variable or signal.

Branch: A branch is a directed line segment joining two nodes.

Transmittance: It is the gain between two nodes.

Input node: A node that has only outgoing branche(s). It is also, called as source and corresponds to independent variable.

Output node: A node that has only incoming branches. This is also called as sink and corresponds to dependent variable.

Path: A path is a traversal of connected branches in the direction of branch arrow.

Loop: A loop is a closed path.

Self loop: It is a feedback loop consisting of single branch.

Loop gain: The loop gain is the product of branch transmittances of the loop.

Nontouching loops: Loops that do not posses a common node. Forward path: A path from source to sink without traversing an node more than once.

Feedback path: A path which originates and terminates at the same node. Forward path gain: Product of branch transmittances of a forward path.

Properties of Signal Flow Graphs:

1) Signal flow applies only to linear systems.

2) The equations based on which a signal flow graph is drawn must be algebraic equations in the form of effects as a function of causes. Nodes are used to represent variables. Normally the nodes are arranged left to right, following a succession of causes and effects through the system.

3) Signals travel along the branches only in the direction described by the arrows of the branches.

4) The branch directing from node Xk to Xj represents dependence of the variable Xj on Xk but not the reverse.

5) The signal traveling along the branch Xk and Xj is multiplied by branch gain akj and signal akjXk is delivered at node Xj.

Guidelines to Construct the Signal Flow Graphs: The signal flow graph of a system is constructed from its describing equations, or by direct reference to block diagram of the system. Each variable of the block diagram becomes a node and each block becomes a branch. The general procedure is

1) Arrange the input to output nodes from left to right.

2) Connect the nodes by appropriate branches.

3) If the desired output node has outgoing branches, add a dummy node and a unity gain branch.

4) Rearrange the nodes and/or loops in the graph to achieve pictorial clarity.

Algebra Addtion rule The value of the variable designated by a node is equal to the sum of all signals entering the node.

Transmission rule The value of the variable designated by a node is transmitted on every branch leaving the node.

Multiplication rule A cascaded connection of n-1 branches with transmission functions can be replaced by a single branch with new transmission function equal to the product of the old ones.

Masons Gain Formula The relationship between an input variable and an output variable of a signal flow graph is given by the net gain between input and output nodes and is known as overall gain of the system. Masons gain formula is used to obtain the over all gain (transfer function) of signal flow graphs.

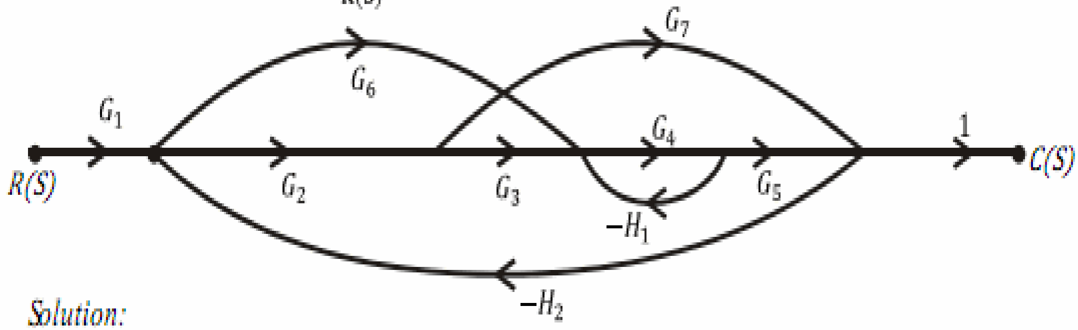
Gain P is given by $P = \frac{1}{\Delta}\sum_k P_k \Delta_k$

Where, P_k is gain of kth forward path, Δ is determinant of graph

Δ=1-(sum of all individual loop gains)+(sum of gain products of all possible combinations of two nontouching loops – sum of gain products of all possible combination of three nontouching loops) + ···

Δ_k is cofactor of kth forward path determinant of graph with loops touching kth forward path. It is obtained from Δ by removing the loops touching the path P_k.

(1) Obtain the closed loop TF, $\frac{C(S)}{R(S)}$ by using Mason's gain formula.



Solution:

Mason's gain formula is, $T = \frac{1}{\Delta}\sum P_K \Delta_K$

No. of forward paths:

$P_1 = G_1 G_2 G_3 G_4 G_5$ $\qquad \Delta_1 = 1$

$P_2 = G_1 G_6 G_4 G_5$ $\qquad \Delta_2 = 1$

$P_3 = G_1 G_2 G_7$ $\qquad \Delta_3 = 1 - \{-H_1 G_4\} = 1 + G_4 H_1$

No. of individual loops:

$L_{11} = -G_4 H_1$

$L_{21} = -G_2 G_3 G_4 G_5 H_2$

$L_{31} = -G_2 G_7 H_2$

$L_{41} = -G_6 G_4 G_5 H_2$

Gain Products of all possible combinations of two non-touching loops:

$L_{12} = (L_{11})(L_{31})$

$= H_1 H_2 G_2 G_4 G_7$

No. of three non-touching loops = 0.

$\Rightarrow \quad \Delta = 1 - \{L_{11} + L_{21} + L_{31} + L_{41}\} + L_{12}$

$$T = \frac{1}{1 - \{L_{11} + L_{21} + L_{31} + L_{41}\} + L_{12}} \times \{P_1\Delta_1 + P_2\Delta_2 + P_3\Delta_3\}$$

$$\therefore \quad \boxed{T = \frac{G_1 G_2 G_3 G_4 G_5 + G_1 G_6 G_4 G_5 + G_1 G_2 G_7[1 + G_4 H_1]}{1 + G_4 H_1 + G_2 G_3 G_4 G_5 H_2 + G_2 G_7 H_2 + G_6 G_4 G_5 H_2 + H_1 H_2 G_2 G_4 G_7}}$$